THE BRAIDING OF
LONG COLUMN-CODED
REGULAR KNOTS

SUPPLEMENT to Pamphlet No.7

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Long Column-Coded Regular Knots

Long column-coded regular knots are knots in which the number of parts exceeds the number of bights considerably. For such knots the conventional **algorithm-diagram** as discussed in Pamphlet No. 7 would become excessively long. Moreover, during the braiding process it might become difficult to keep track of the position in such a long algorithm-diagram. The same can be said for the conventional half-cycle algorithms.

In order to overcome these problems, we use for long column-coded regular knots a modified form of algorithm-diagram which we shall discuss in this supplement with the aid of examples.

Example 1: $p = 101$, $b = 8$.

The **complementary cyclic bight-number scheme** is determined as usual (see Pamphlet No. 7, pgs. 14 – 15); hence we obtain here the sequence 0 , 3 , 6 , 1 , 4 , 7 , 2 , 5 (a total of 8 bight-numbers since $b = 8$).

Since $p = 101$ and $b = 8$, we obtain $101 \div 8 = 12$ with remainder 5. Thus in the conventional construction of the algorithm-diagram, the sequence 0 , 3 , 6 , 1 , 4 , 7 , 2 , 5 would be repeated 12 times, and finally the part sequence 0 , 3 , 6 , 1 , 4 would complete the total sequence of bight-numbers for the Left to Right half-cycles.

In order to alleviate the problems mentioned above, associated with such a long conventional algorithm-diagram, we modify the construction of the algorithm-diagram. We split the conventional algorithm-diagram (a row of 102 cells) into sections, each b cells long, and stack these sections as rows, from top to bottom in a table. This table consists then of b columns, hence 8 columns in our example, and results as shown on page 2.

For the Left to Right half-cycles, the columns are numbered from left to right according to the complementary cyclic bight-number scheme. Below this numbered line we draw $n + 1$ rows, where n is obtained from the formula $p = n \cdot b + r$. Hence in our case we draw 13 rows ($101 = 12 \times 8 + 5$, thus $n = 12$). Note that each row contains $b = 8$ cells.

The first cell in the first row always remains empty since it represents the left-hand bight-boundary of the knot. Then follow $p - 1 = 100$ cells which represent the crossing-columns of the knot. For the Left to Right half-cycles, the rows are read from **left** to **right**, and in consecutive order from the top of the table downwards.

In the last row, the first $r = 5$ cells represent the last $r = 5$ crossing-columns on a Left to Right half-cycle. The $(r + 1)^{th} = 6^{th}$ cell in the last row represents the right-hand bight-boundary of the knot.

Below the last row in the table we set off the complementary cyclic bight-number scheme from **right** to **left**, starting with 0 for the $(r + 1)^{th} = 6^{th}$ column from the left.

For the Right to Left half-cycles, the rows are read from **right** to **left**, and in consecutive order from the bottom of the table upwards.

The length-wise centre of the knot is indicated in the table by a thick vertical line segment; here this length-wise centre is between the 3^{rd} and the 4^{th} cell of row 7 (note that there are $(p - 1)/2 = 50$ intersection-column cells to either side of the length-wise numbers for this half-cycle, from all the rows, taken in top to bottom order (and vice-versa for a *right* bight-boundary to a *left* bight-boundary pass). For more complicated coding patterns (such as in Examples 3 and 4 below) braiding directly from these modified algorithm-diagrams is greatly simplified by this type of consecutive "row-braiding". The numbering of the rows facilitates keeping track of the position as the braiding proceeds.

As mentioned in the beginning, the conventional half-cycle algorithms for long knots become unwieldily long, and consequently it may become difficult during braiding to keep track of the braiding position within a half-cycle. The tabular method just given obviates this problem. Some braiders, however, might still prefer using half-cycle algorithms. For them we shall, for each Example knot, present a simplified set of half-cycle algorithms as an alternative to the conventional ones. The new notation introduced should become clear from the four Examples discussed in this Supplement.

For Example 1 ($p = 101$, $b = 8$), the simplified half-cycle algorithms are as follows:

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run .
2. $(i = 0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $12o$.
3. $(i = 0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $12o$.
4. $(i = 1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $25o$.
5. $(i = 1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $25o$.
6. $(i = 2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $37o$.
7. $(i = 2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $37o$.
8. $(i = 3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $50o$.
9. $(i = 3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $50o$.
10. $(i = 4)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2o - u - \{4o - u\}12\times$.
11. $(i = 4)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2o - u - \{4o - u\}12\times$.
12. $(i = 5)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $2o - u - \{o - u - 3o - u\}12\times$.
13. $(i = 5)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $2o - u - \{o - u - 3o - u\}12\times$.
14. $(i = 6)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $\{o - u\}2\times - \{o - u - 2o - u - o - u\}12\times$.
15. $(i = 6)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $\{o - u\}2\times - \{o - u - 2o - u - o - u\}12\times$.
16. $(i = 7)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o - u - \{o - 2u - o - u - 2o - u\}12\times - o - u$.

Example 2: $p = 101$, $b = 7$.

The **complementary cyclic bight-number scheme** is again determined as usual (see Pamphlet No. 7, pgs. 14 – 15); hence we obtain the sequence 0, 2, 4, 6, 1, 3, 5 (a total of 7 bight-numbers since $b = 7$).

Since $p = 101$ and $b = 7$, we obtain $101 \div 7 = 14$ with remainder 3.

Again, draw a table with b columns, hence in this case a table with 7 columns as shown on page 4. For the Left to Right half-cycles the columns are numbered from left to right according to the complementary cyclic bight-number scheme. Below this numbered line we draw $n + 1$ rows, where n is obtained from the formula $p = n \cdot b + r$. Hence in this case we draw 15 rows ($101 = 14 \times 7 + 3$, thus $n = 14$). Note that each row contains $b = 7$ cells.

The first cell in the first row always remains empty since it represents the left-hand bight-boundary of the knot. Then follow $p - 1 = 100$ cells which represent the crossing-columns of the knot. For the Left to Right half-cycles, the rows are read from **left** to **right**, and in consecutive order from the top of the table downwards.

In the last row, the first $r = 3$ cells represent the last $r = 3$ crossing-columns on a Left to Right half-cycle. The $(r + 1)^{th} = 4^{th}$ cell in the last row represents the hand bight-boundaries are obtained (see the diagram at the right of the bottom of the table). Furthermore, it is important that the first half-cycle is properly laid down with the correct number of turns around the core (this number is equal to $p/(2b) = 101/14 = 7\frac{3}{14}$), and that the crossings between the second and first half-cycles are properly positioned.

Although braiding directly from the above obtained algorithm-diagram is undoubtedly the easiest, for those who prefer to braid according to half-cycle algorithms we present simplified half-cycle algorithms for this example thus:

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run.
2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $14o$.
3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $14o$.
4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $28o$.
5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $28o$.
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $43o$.
7. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $43o$.
8. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $30o - u - \{3o - u\}6\times - 2o$.
9. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $30o - u - \{3o - u\}6\times - 2o$.
10. $(i=4)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o - u - \{4o - u\}7\times - \{o - u - 2o - u\}7\times$.
11. $(i=4)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o - u - \{4o - u\}7\times - \{o - u - 2o - u\}7\times$.
12. $(i=5)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o - u - \{2o - u\}14\times - \{o - 2u - 2o - u\}7\times$.
13. $(i=5)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o - u - \{2o - u\}14\times - \{o - 2u - 2o - u\}7\times$.
14. $(i=6)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o - \{2u - 2o - u - 2o\}7\times - \{2u - o - 2u - 2o\}7\times - u$.

The long knot of this example can of course also readily be braided by the weaving pattern method since $|2p|_b = b - 1$ (see Pamphlet No. 7, pgs. 32–33).

Now we have seen, with the aid of two examples, how an algorithm-diagram for a long column-coded regular knot is constructed, let us investigate some relationships which exist within this type of algorithm-diagram.

With our earlier notation, let the number of parts of a long column-coded regular knot be $p = n \cdot b + r$; hence $|p|_b = r$.
Then:

1. For b **even**, r must be **odd** (p and b must be coprime, hence p must be odd). There are no columns in the algorithm-diagram which have the same bight-number for both the Right to Left and the Left to Right half-cycles.

2. For b **odd** and r **odd**, the column $\left(\frac{b+r}{2} + 1\right)$ from the left in the algorithm-diagram carries for both the Right to Left and the Left to Right half-cycles the bight-number $\left|\left(\frac{b+r}{2}\right)\Delta^*\right|_b = \frac{|p \cdot \Delta^*|_b}{2} = \frac{|r \cdot \Delta^*|_b}{2} = \frac{b-1}{2}$, where Δ^* is calculated with the formulae as given on pg. 15 of Pamphlet No. 7, or with the formula:

$$\Delta^* = \frac{n^* \cdot b - 1}{r} = \frac{n^* \cdot b - 1}{|p|_b},$$

where n^* is the smallest positive integer so that $(n^* \cdot b - 1)$ is a multiple of r. Note that in general p can be odd or even, and that only when p is odd, the knot has in braiding terms a length-wise centre.

(i) When the number of bights is even, the maximum initial number of half-cycles which can be laid down without interweaving is equal to $(b+1)$. Hence the maximum number of columns in the algorithm-diagram with a constant coding per column is equal to b.
(ii) When the number of bights is **odd**, the maximum initial number of half-cycles which can be laid down without interweaving is equal to b. Hence the maximum number of columns in the algorithm-diagram with a constant coding per column is equal to $(b-1)$.

If the initial number of half-cycles which are laid down without interweaving is equal to 1, then the algorithm-diagram has no columns with constant coding.

If the initial number of half-cycles which are laid down without interweaving is equal to 3, then the algorithm-diagram has 2 columns with constant coding. These columns carry the bight-numbers 0 and $(b-1)$.

If the initial number of half-cycles which are laid down without interweaving is equal to 5, then the algorithm-diagram has 4 columns with constant coding. These columns carry the bight-numbers $0, 1$ and $(b-1), (b-2)$.

If the initial number of half-cycles which are laid down without interweaving is equal to 7, then the algorithm-diagram has 6 columns with constant coding. These columns carry the bight-numbers $0, 1, 2$ and $(b-1), (b-2), (b-3)$.

If the initial number of half-cycles which are laid down without interweaving is equal to x (x must be *odd*), then the algorithm-diagram has $(x-1)$ columns with constant coding. These columns carry the bight-numbers $0, 1, 2, \cdots, \frac{x-5}{2}, \frac{x-3}{2}$ and $(b-1), (b-2), (b-3), \cdots, \left(b-\frac{x-3}{2}\right), \left(b-\frac{x-1}{2}\right)$.

The codings we have chosen for the examples 1 and 2 above are special ones. They are not only length-wise symmetric relative to the length-wise centre of the knot, but they furthermore ensure that initially the maximum number of half-cycles without interweaving can be laid down. Such conditions speed up the construction of a knot considerably, and also decrease wear and tear on the string.

The long column-coded regular knots of the following two examples were designed by American braiders: the knot in Example 3 was designed by Ed Pass of Florence, Arizona, and was used in his quirt for the "Cowboy Classics"; the knot in Example 4 (a variation of the design of Example 3) was designed by Jim Barcus of Mesa, Arizona. Both these knots can readily be braided according to their weaving patterns, since for the long knot of example 3 the value of $|p|_b = 1$, and for the long knot of example 4 the value of $|p|_b = b-1$ (see Pamphlet No. 7, pgs. 32–33). However braiding these long knots directly from their modified algorithm-diagrams is certainly easier, and hence we shall discuss their algorithm-diagrams below.

Example 3: $p = 137$, $b = 8$.

The **complementary cyclic bight-number scheme** is determined as usual (see Pamphlet No. 7, pgs. 14–15); hence the sequence 0, 7, 6, 5, 4, 3, 2, 1 (a total of 8 bight-numbers since $b = 8$) is obtained for this knot.

Since $p = 137$ and $b = 8$, we obtain $137 \div 8 = 17 +$ remainder of 1.

right-hand bight-boundary of the knot.

Below the last row in the table we set off the complementary cyclic bight-number scheme from **right** to **left**, starting with 0 for the $(r+1)^{th} = 2^{nd}$ column from the left.

For the Right to Left half-cycles, the rows are read from **right** to **left**, and in consecutive order from the bottom of the table upwards.

The length-wise centre of the knot is in the table again indicated by a thick vertical line segment; here this length-wise centre is between the 5^{th} and the 6^{th} cell of row 9.

A coding is chosen which makes a pleasing pattern, although it is not length-wise symmetrical relative to the length-wise centre of the knot. However, the chosen coding eliminates interweaving for the first 3 half-cycles. Hence interweaving does not start until the 4^{th} half-cycle. Thus out of the 16 half-cycles we have 13 half-cycles in which interweaving takes place.

Again it should be stressed that in order to eliminate or at least minimize final adjustments after the knot has been braided, it is important not only to indicate on the core the position of the length-wise centre of the knot, but also to ensure that the proper relative positions of the bight index-numbers on the left-hand and right-hand bight-boundaries are obtained (see the diagram at the right of the bottom of the table). Furthermore, it is important that the first half-cycle is again properly laid down with the correct number of turns around the core (this number is equal to $p/(2b) = 137/16 = 8\frac{9}{16}$), and that the crossings between the second and first half-cycles are properly positioned.

Although braiding directly from the above obtained algorithm-diagram by the consecutive "row-braiding" method (refer pg. 3) is undoubtedly the easiest, and is here in fact closely akin to braiding according to a weaving pattern. Braiding this knot according to half-cycle algorithms is a 'little' cumbersome; even in their simplified forms they are too long for easy braiding. Nevertheless, for those who insist on braiding according to half-cycle algorithms we present the simplified half-cycle algorithms for this example thus:

1. $\mathrm{L} \longrightarrow \mathrm{R}$: free run.
2. $(i=0)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $17o$.
3. $(i=0)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $17o$.
4. $(i=1)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $\{u-o\}2\times - u - 5o - u - 3o - u - o - u - 5o - \{u-o\}6\times$.
5. $(i=1)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $\{u-o\}5\times - u - 5o - u - 3o - u - o - u - 5o - \{u-o\}3\times$.
6. $(i=2)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $o - \{u-2o\}2\times - u - o - u - 2o - u - 3o - u - 5o - u -$
   $2o - u - o - u - 2o - u - 3o - \{u-2o\}5\times - u - o$.
7. $(i=2)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $o - \{u-2o\}5\times - u - o - u - 2o - u - 3o - u - 5o - u -$
   $2o - u - o - u - 2o - u - 3o - \{u-2o\}2\times - u - o$.
8. $(i=3)$ $\mathrm{R} \longrightarrow \mathrm{L}$: $\{u-o\}3\times - u - 3o - u - 2o - u - 3o - u - 4o -$
   $\{u-o-u-3o\}2\times - u - 2o - u - 3o - u - 4o -$
   $[\{u-o\}2\times - u - 3o]2\times - \{u-o\}3\times$.
9. $(i=3)$ $\mathrm{L} \longrightarrow \mathrm{R}$: $u - o - u - 3o - [\{u-o\}2\times - u - 3o]2\times -$
   $u - 2o - u - 3o - u - 4o - \{u-o-u-3o\}2\times -$
   $u - 2o - u - 3o - u - 4o - u - 3o - \{u-o\}3\times$.

In the last row, the first $r = 7$ cells represent the last $r = 7$ crossing-columns on a Left to Right half-cycle. The $(r+1)^{th} = 8^{th}$ cell in the last row represents the right-hand bight-boundary of the knot.

Below the last row in the table we set off the complementary cyclic bight-number scheme from right to left, starting with 0 for the $(r+1)^{th} = 8^{th}$ column from the left.

For the Right to Left half-cycles, the rows are read from right to left, and in consecutive order from the bottom of the table upwards.

The length-wise centre of the knot is in the table again indicated by a thick vertical line segment; here this length-wise centre is between the 4^{th} and the 5^{th} cell of row 9.

Besides the chosen coding being length-wise unsymmetric relative to the length-wise centre of the knot, it has only the first half-cycle (the free run) without interbraiding. Hence interweaving starts with the 2^{nd} half-cycle. Thus out of the 16 half-cycles we have 15 half-cycles in which interweaving takes place.

L → R

	0	1	2	3	4	5	6	7
1		\	/	\	/	\	/	\
2	/	\	/	\	\	/	/	\
3	/	\	/	\	/	\	/	\
4	/	\	/	\	\	/	/	\
5	/	\	/	\	/	\	/	\
6	/	\	/	\	\	/	/	\
7	/	\	\	/	\	/	\	\
8	/	/	\	/	\	/	\	\
9	/	\	/	/	\	\	/	\
10	/	\	/	\	/	\	\	\
11	/	/	/	\	/	\	/	\
12	/	\	/	/	\	\	/	\
13	/	/	\	/	\	/	\	\
14	/	/	\	/	\	/	\	\
15	/	\	/	/	\	\	/	\
16	/	\	/	\	\	/	\	/
17	\	/	\	/	\	/	\	
	7	6	5	4	3	2	1	0

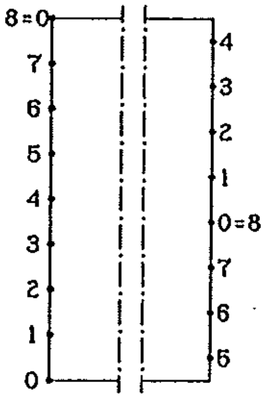
L ← R

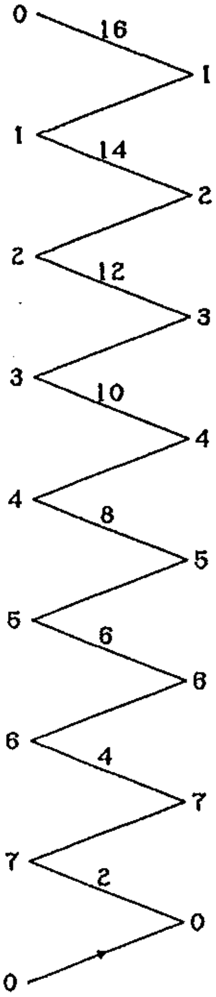
p=135 b=8

$|-p|_b = |-135|_8 = 1$

$m^* = -1$ $\Delta^* = 1$

$|p|_b = |135|_8 = 7$





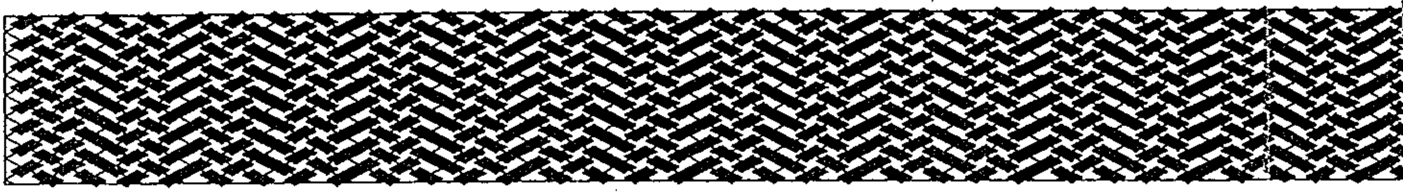
13. $(i=5)$ $\mathrm{L}\longrightarrow\mathrm{R}$: $[\{u-o\}4\times-2u-2o]3\times-2u-2o-2u-o-u-3o-u-o-$
$\{u-2o\}2\times-2u-\{o-u\}3\times-3o-\{u-o\}2\times-$
$u-2o-2u-2o-u-o-u-3o-u-o-\{u-2o\}2\times-$
$2u-o-u-o-2u-\{o-u\}3\times-o\,.$

14. $(i=6)$ $\mathrm{R}\longrightarrow\mathrm{L}$: $\{o-u\}2\times-o-2u-o-\{u-2o\}3\times-2u-3o-$
$\{u-o\}2\times-u-2o-\{u-o\}3\times-u-2o-2u-2o-$
$\{u-o\}2\times-2u-3o-u-o-\{u-2o\}2\times-2u-3o-$
$\{u-o\}2\times-u-2o-u-o-u-3o-2u-\{2o-u\}2\times-$
$\{o-u\}2\times-[2o-2u-\{2o-u\}2\times-\{o-u\}2\times]2\times-o\,.$

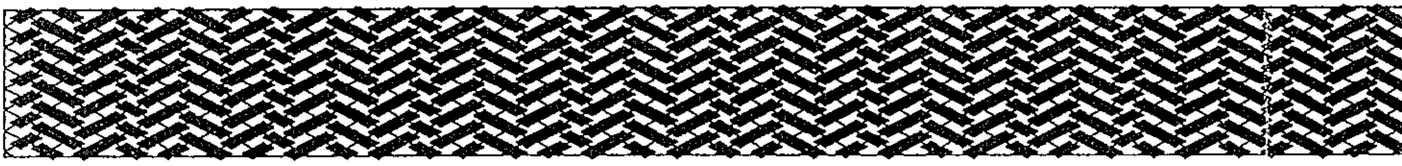
15. $(i=6)$ $\mathrm{L}\longrightarrow\mathrm{R}$: $\{u-o\}2\times-[u-2o-u-o-2u-3o-\{u-o\}2\times]2\times-$
$u-2o-u-o-2u-3o-2u-\{o-u\}2\times-2o-$
$\{u-o\}3\times-u-2o-2u-2o-\{u-o\}2\times-$
$2u-3o-u-o-\{u-2o\}2\times-2u-3o-\{u-o\}2\times-$
$u-2o-\{u-o\}3\times-u-2o-2u-2o-u-o-$
$\{2u-o\}2\times-\{u-o\}2\times-u\,.$

16. $(i=7)$ $\mathrm{R}\longrightarrow\mathrm{L}$: $\{o-u\}5\times-2o-\{u-o\}2\times-u-2o-2u-o-u-$
$[2o-\{u-o\}2\times-2u]2\times-o-u-2o-2u-$
$\{o-u\}3\times-o-3u-3o-\{u-o\}3\times-$
$u-2o-2u-o-u-2o-\{u-o\}2\times-$
$2u-2o-u-o-u-2o-u-o-$
$[2u-2o-\{u-o\}6\times]2\times-2u-2o-\{u-o\}5\times\,.$

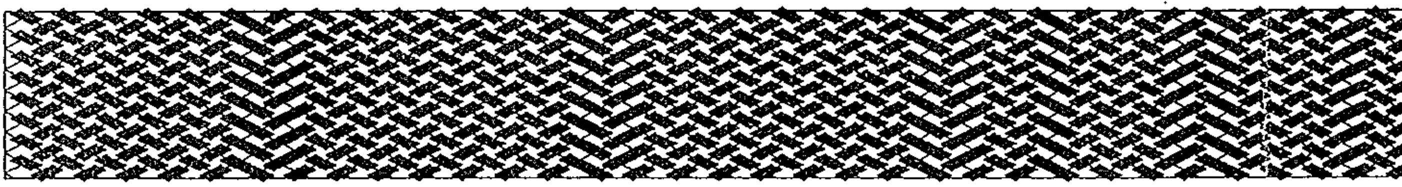
(Continued on p.15)



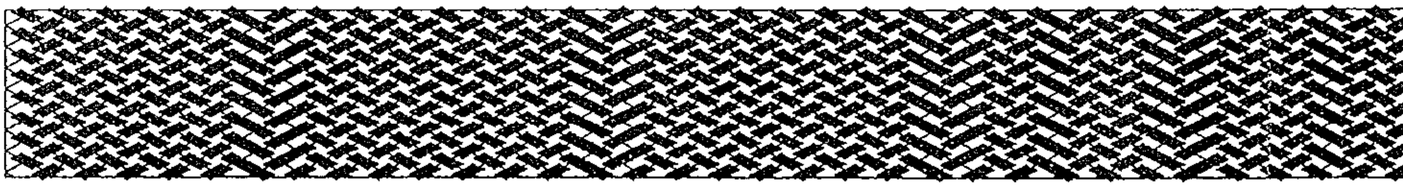
EXAMPLE 1. LONG COLUMN-CODED REGULAR KNOT WITH p=101 AND b=8



EXAMPLE 2. LONG COLUMN-CODED REGULAR KNOT WITH p=101 AND b=7



EXAMPLE 3. LONG COLUMN-CODED REGULAR KNOT WITH p=137 AND b=8
DESIGNED BY ED PASS (FLORENCE. U.S.A.)
INCORPORATED IN HIS QUIRT FOR THE 'COWBOY CLASSICS'



EXAMPLE 4. LONG COLUMN-CODED REGULAR KNOT WITH p=135 AND b=8
DESIGNED BY JIM BARCUS (MESA. U.S.A.)